$$\mathcal{E}_{me}^{(2)} = \frac{1}{2} B_{111} e^2 (n_1^4 \alpha_1^2 + n_2^4 \alpha_2^2 + n_3^4 \alpha_3^2)$$

$$+ B_{123} e^2 (n_1^2 n_2^2 \alpha_3^2 + n_2^2 n_3^2 \alpha_1^2 + n_3^2 n_1^2 \alpha_2^2)$$

$$+ 2B_{144} e^2 (n_1^2 n_2 n_3 \alpha_2 \alpha_3 + n_2^2 n_3 n_1 \alpha_3 \alpha_1 + n_3^2 n_1 n_2 \alpha_1 \alpha_2) +$$

$$2B_{441} e^2 (n_2^2 n_3^2 \alpha_1^2 + n_3^2 n_1^2 \alpha_2^2 + n_1^2 n_2^2 \alpha_3^2)$$

$$+ 2B_{155} e^2 ((n_1^2 + n_2^2) n_1 n_2 \alpha_1 \alpha_2 + (n_2^2 + n_3^2) n_2 n_3 \alpha_2 \alpha_3$$

$$+ (n_3^2 + n_1^2) n_3 n_1 \alpha_3 \alpha_1)$$

$$+ 4B_{456} e^2 (n_3^2 n_1 n_2 \alpha_1 \alpha_2 + n_1^2 n_2 n_3 \alpha_2 \alpha_3 + n_2^2 n_3 n_1 \alpha_3 \alpha_1)$$

$$+ \theta(e^3) + \ldots$$

Averaging this expression with the aid of Table 1 and neglecting a function of strain only gives the second order magnetoelastic energy correct to second order in e.

$$\mathcal{E}_{me}^{(2)} = \frac{1}{35} (6B_{111} - 2B_{123} + 3B_{144} + 18B_{155} - 4B_{441} + 6B_{456}) e^2 \cos^2\theta$$

The total average magnetoelastic energy consistent with the interacting grain theory correct to second order in e is

$$\mathcal{E}_{me} = \left[\left(\frac{2}{5} b_1 + \frac{3}{5} b_2\right) e + \left(\frac{14}{10} b_1 + \frac{11}{10} b_2 + \frac{6}{35} B_{111} - \frac{2}{35} B_{123} + \frac{3}{35} B_{144} + \frac{18}{35} B_{155} - \frac{4}{35} B_{441} + \frac{6}{35} B_{456}\right) e^2\right] \cos^2\theta. \qquad \text{(III.5)}$$

III.3. Finite Strain Correction to Independent Grain Theory

The independent grain theory requires solutions of the <100> problem and the <111> problem from finite strain theory. For uniaxial strain along a <100> direction, the magnetoelastic energy reduces to

$$\mathcal{E}_{me}^{<100>} = b_1 E_{11} \alpha_1^{*2} + \frac{1}{2} B_{111} E_{11}^2 \alpha_1^{*2} .$$

Using

$$E_{11} = e + \frac{e^2}{2} \tag{III.6}$$

and

$$\alpha_1^{*2} = (1 + 2e)\alpha_1^2 + \theta(e^2) + \ldots , \tag{III.7}$$

one obtains

$$\mathcal{E}_{me}^{<100>} = \left[b_1 e + \left(\frac{5}{2} b_1 + \frac{B_{111}}{2} \right) e^2 \right] \alpha_1^2 \tag{III.8}$$

correct to second order in e.

The solution of the <111> problem is somewhat more difficult. One method is to rotate the first and second order magnetoelastic tensors (fourth and sixth rank tensors, respectively) to a coordinate system coincident with the <111> crystal axes. In this system,

$$\mathcal{E}_{me}^{<111>} = b'_{11} E_{11} \alpha_1^{*2} + b'_{12} E_{11} (\alpha_2^{*2} + \alpha_3^{*2}) + \frac{1}{2} B'_{111} E_{11}^2 \alpha_1^{*2} + \frac{1}{2} B'_{112} E_{11}^2 (\alpha_2^{*2} + \alpha_3^{*2})$$

where

$$b'_{11} = \frac{1}{3} b_1 + \frac{2}{3} b_2 ,$$

$$b'_{12} = \frac{1}{3} b_1 - \frac{1}{3} b_2 ,$$